

A SUSY SO(10) Model with Large $\tan\beta$

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Abstract

We construct a supersymmetric SO(10) model with the asymptotic relation $\tan\beta \simeq m_t/m_b$ automatically arising from its structure. The model retains the significant Minimal Supersymmetric Standard Model predictions for $\sin^2\theta_w$ and α_s and contains an automatic Z_2 matter parity. Proton decay through $d = 5$ operators is sufficiently suppressed. It is remarkable that no global symmetries need to be imposed on the model.

The renormalization group equations of the minimal supersymmetric standard model (MSSM) with a supersymmetry breaking scale $M_s \sim 1$ TeV are remarkably consistent⁽¹⁾ with the measured values of $\sin^2\theta_w$ and α_s and unification of the three gauge couplings at a scale $\sim 10^{16}$ GeV. This significant property certainly makes the MSSM more attractive than the non-supersymmetric standard model in spite of the larger number of undetermined parameters that it contains. An important undetermined new parameter introduced by supersymmetry, known as $\tan\beta$, is the ratio of the vacuum expectation values (vevs) of the electroweak higgs doublets $h^{(1)}$ and $h^{(2)}$ giving mass to the up-type quarks and the down-type quarks and charged leptons respectively. $\tan\beta$ remains undetermined if MSSM is embedded in the minimal supersymmetric (SUSY) SU(5) model. In contrast, embeddings in supersymmetric grand unified theories (SUSY GUTs) based on larger gauge groups may lead⁽²⁾ to the asymptotic relation $\tan\beta \simeq m_t/m_b$. In a previous paper⁽³⁾, we formulated widely applicable conditions under which this asymptotic relation holds. A central role in these conditions is played by the $SU(2)_R$ gauge symmetry. Moreover, we constructed specific SUSY GUTs, mostly based on semisimple gauge groups, where these conditions apply. The only simple gauge group briefly discussed was SO(10). However, this discussion did not intend to produce a specific SO(10) model but only to indicate the applicability of our conditions.

The purpose of the present paper is to construct a specific SUSY SO(10) model where the asymptotic relation $\tan\beta \simeq m_t/m_b$ is an automatic consequence of the structure of the theory with the usual MSSM predictions for $\sin^2\theta_w$ and α_s being retained. Two problems closely related to the predic-

tion of large $\tan\beta$ in this context are the gauge hierarchy problem and the problem of proton decay proceeding through $d = 5$ operators⁽⁴⁾. Here we do not attempt to provide the mechanism forcing one pair of electroweak higgs doublets to remain light but we are satisfied if the relation $\tan\beta \simeq m_t/m_b$ holds independently of the reason for which one such pair remains light. As far as the proton decay problem is concerned, we simply aim at loosening the usual tight relationship of minimal SUSY GUTs between colour triplet higgsino mediated proton decay amplitude and light fermion masses. Because of this relationship the proton decay amplitude mediated by colour triplet higgsino exchange is marginally compatible with the present experimental bound on proton lifetime. Moreover, large values of $\tan\beta$ implied by the asymptotic relation $\tan\beta \simeq m_t/m_b$ make the situation even worse. Any viable SUSY SO(10) model with large $\tan\beta$ must therefore be equipped with a mechanism which suppresses the $d = 5$ operators relevant for proton decay mediated by colour triplet higgsino exchange. The usual mechanism⁽⁵⁾ for achieving such a suppression involves arranging for the colour triplets which belong to the same SO(10) multiplets with the electroweak higgs doublets to acquire masses close to the unification scale but with mass partners having suppressed couplings to the ordinary quarks and leptons. This arrangement, in most cases, requires non-minimal field content, additional discrete or continuous global symmetries and, sometimes, even a mild tuning of parameters. For our purposes significant departure from minimality is certainly undesirable because it reduces the predictability of the model. However, we succeed here in implementing the above described mechanism with a minimal enlargement of the field content of the theory, with only a

mild tuning of parameters (by an order of magnitude or so) and with the asymptotic relation $\tan\beta \simeq m_t/m_b$ being automatically guaranteed. We find particularly worth emphasizing the fact that we do not have to recourse to the imposition of any discrete symmetries since all the above achievements are immediate consequences of the gauge symmetry $\text{SO}(10)$ and the choice of the field content.

It is quite remarkable that even the discrete matter parity, necessary for suppressing the rapid proton decay through $d = 4$ operators, could be an automatic consequence of the structure of the theory. Any $\text{SO}(10)$ model has an obvious Z_2 symmetry under which the spinors change sign with the tensors remaining invariant. If no spinor acquires a vev this Z_2 symmetry remains unbroken and plays the role of the matter parity provided the three light generations belong to three 16's of $\text{SO}(10)$. It is easily seen that the Z_2 matter parity just described is a subgroup of both $U(1)_{B-L}$ and of the Z_4 center of $\text{SO}(10)$.

We consider a SUSY $\text{SO}(10)$ model with the ordinary quarks and leptons belonging as usual to three 16's denoted by ψ . The $\text{SO}(10)$ breaks directly down to $SU(3)_c \times SU(2)_L \times U(1)_Y$. This is achieved by using one superfield in each of the 126, $\overline{126}$, 45 and 54 representations of $\text{SO}(10)$. We denote these fields as χ_{ijklm} , $\bar{\chi}_{ijklm}$, ϕ_{ij} and η_{ij} respectively. Note that $\chi_{ijklm}(\bar{\chi}_{ijklm})$ is a fully antisymmetric and (anti)self-dual $\text{SO}(10)$ tensor whereas $\phi_{ij}(\eta_{ij})$ is an antisymmetric (symmetric and traceless) $\text{SO}(10)$ tensor. The superpotential terms allowed by the $\text{SO}(10)$ symmetry are $\bar{\chi}_{ijklm}\chi_{ijklm}$, $\phi_{ij}\bar{\chi}_{iklmn}\chi_{jklmn}$, $\eta_{ij}\chi_{iklmn}\chi_{jklmn}$, $\eta_{ij}\bar{\chi}_{iklmn}\bar{\chi}_{jklmn}$, $\phi_{ij}\phi_{ij}$, $\eta_{ij}\eta_{ij}$, $\eta_{ij}\eta_{jk}\eta_{ki}$ and $\eta_{ij}\phi_{jk}\phi_{ki}$. It can be shown⁽⁶⁾ that, with these fields, the $\text{SO}(10)$ breaking proceeds without

leaving any accidental pseudogoldstone particles. Thus, all particles in this sector of the theory can be given masses close to the unification scale. This fact is important for retaining the successful MSSM predictions for $\sin^2\theta_w$ and α_s .

In order to achieve the electroweak breaking in a way that allows us to implement the above mentioned mechanism for suppressing proton decay through $d = 5$ operators, we introduce two more superfields, one in the 10 and one in the $210'$ representation of $SO(10)$. These fields are denoted as ζ_i and θ_{ijk} respectively. ζ_i is a $SO(10)$ vector and θ_{ijk} is a fully symmetric and traceless $SO(10)$ tensor. Both ζ and θ contain components which have the right standard model quantum numbers to contribute to the electroweak doublets but only ζ has yukawa couplings to the ψ 's which contain the ordinary quarks and leptons. The fields ζ and θ also contain colour triplets and antitriplets. So, if one succeeds in arranging for the colour triplets and antitriplets in ζ to have as mass partners mainly the colour antitriplets and triplets in θ respectively, one can suppress the $d = 5$ operators for proton decay. The relevant mass term is provided by the superpotential term $\lambda\zeta_i\theta_{ijk}\eta_{jk}$ and must be the dominant mass term between these colour triplets and antitriplets for this mechanism to work. The other mass terms between these triplets and antitriplets provided by the superpotential terms $M\zeta_i\zeta_i$, $\lambda'\eta_{ij}\zeta_i\zeta_j$, $M'\theta_{ijk}\theta_{ijk}$ and $\lambda''\eta_{ij}\theta_{ikt}\theta_{jkl}$ must be slightly suppressed (by an order of magnitude or so). An explicit calculation of the mass matrix between the colour triplets and antitriplets in ζ and θ which belong to the (6,1,1) representation of $G_{422} \equiv SU(4)_c \times SU(2)_L \times SU(2)_R$ gives $M + \lambda'\eta_o/6$, $M' - \lambda''\eta_o/54$ in the main diagonal and $\lambda\eta_o/3\sqrt{2}$ in the off-diagonal entries. Here η_o is defined by

the vev of $\eta_{ij} : \langle \eta_{ij} \rangle = \eta_o \text{diag} (1/6, \dots, 1/6, -1/4, \dots, -1/4)$, where the $1/6$ appears six times and the $-1/4$ four times. The mass matrix of the electroweak doublets in ζ and θ , which happen to belong to the $(1,2,2)$ representation of G_{422} , consists of $M - \lambda'\eta_o/4$, $M' - \lambda''\eta_o/9$ in the main diagonal and $-\lambda\eta_o/4$ in the off-diagonal entries. One can also compute the masses of the rest of the components of θ . They turn out to be $M' + \lambda''\eta_o/6$, $M' - \lambda\eta_o/9$, $M' - \lambda\eta_o/4$ and $M' + \lambda\eta_o/36$ for the $(50,1,1)$, $(6,3,3)$, $(1,4,4)$ and $(20',2,2)$ components respectively. Here again the decomposition of θ is under G_{422} . The electroweak pair of doublets is chosen to lie in an arbitrary generic linear combination of the doublets in ζ and θ by fine tuning one eigenvalue of the corresponding 2×2 mass matrix. It is then obvious that the diagonal colour (anti)triplet mass terms can be slightly suppressed without affecting any physical masses in the theory. This fact combined with the statement at the end of the previous paragraph means that all the states in the theory except the MSSM states (and the three right-handed neutrinos) can acquire masses close to the unification scale. Consequently, the significant MSSM prediction for $\sin^2\theta_w$ and α_s is retained with the $d = 5$ operators for proton decay being sufficiently suppressed at the same time.

It is important to note that the fields $\bar{\chi}, \chi$ also contain candidate electroweak doublets. Always restricting ourselves to renormalizable superpotential terms we find that these doublets do not couple to the doublets in ζ and θ by mass terms. This is due to the absence of any trilinear superpotential terms coupling ζ or θ with $\bar{\chi}$ or χ and the fields $\bar{\chi}, \chi, \phi$ or η acquiring large vevs. So the doublet sector in ζ and θ does not mix, through renormalizable interactions, with the doublets in $\bar{\chi}, \chi$. Note that, in principle,

we had the option to choose the MSSM pair of higgs doublets from the $\bar{\chi}, \chi$ sector rather than from the ζ, θ sector. We avoided this possibility since we prefer to keep the successful asymptotic relation $m_b \simeq m_\tau$ over the relation $m_b \simeq \frac{1}{3}m_\tau$. Moreover, the mechanism we employed for suppressing proton decay through $d = 5$ operators would be of no use with this choice.

The mass matrix of the electroweak doublets in ζ and θ has an important property: it does not get renormalizable contributions from vevs which break the $SU(2)_R$ (and the $SU(4)_c$) symmetry. Such vevs are acquired by $\bar{\chi}, \chi$ and ϕ but trilinear couplings of ζ or θ with ζ or θ and one of the $\bar{\chi}, \chi$ or ϕ are not allowed by the $SO(10)$ symmetry. This means that the unique pair of electroweak doublets forms a single $SU(2)_R$ -doublet. Corrections from non-renormalizable terms cannot alter this significant property since the orthogonal pair of heavy doublets, which are also $SU(2)_R$ -partners, is not affected by such small contributions.

Since the three ordinary light generations belong entirely to the three ψ 's, it is obvious that the $SU(2)_L$ -doublet quarks and leptons are $SU(2)_R$ -singlets with the $SU(2)_L$ -singlet antiquarks and antileptons being $SU(2)_R$ -doublets. This observation combined with the facts that the unique electroweak higgs doublet pair $h^{(1)}, h^{(2)}$ forms (to a very good approximation) a $SU(2)_R$ -doublet and that the quark and lepton tree-level (Dirac) mass terms must be (to the same approximation) $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ (and even G_{422}) invariant implies that $\tan\beta \simeq m_t/m_b \simeq m_{\nu_\tau}^D/m_\tau$ (and $m_b \simeq m_\tau$). Analogous relations for the first two families are not expected to hold since we anticipate for them substantial corrections to the tree-level masses.

One readily reaches the same conclusion with immediate application of the proposition stated in ref.(3). From the above discussion is obvious that the only states which remain massless in the supersymmetric limit after the breaking of $SO(10)$ are the MSSM states with all other states acquiring super-heavy masses. Also, the unique pair of electroweak doublets forms a $SU(2)_R$ -doublet and the ordinary quarks and leptons are unique under (matter parity) \times (standard model group). Besides the Z_2 matter parity belongs to the center of $SO(10)$. The conditions stated in ref.(3) are then satisfied and, consequently, the approximate relation $\tan\beta \simeq m_t/m_b$ holds. In addition, there are exactly three standard model singlet states which can play to role of the right-handed neutrinos. They are the standard model singlets in the ψ 's. We then also have $\tan\beta \simeq m_{\nu_\tau}^D/m_\tau$ with $m_{\nu_\tau}^D$ being the Dirac mass of the τ -neutrino.

The fields $\bar{\chi}$ and χ also contain colour triplet and antitriplet components which form a (6,1,1) representation of G_{422} . Due to the couplings $f\psi\psi\bar{\chi}$ (and the couplings $f'\bar{\chi}\bar{\chi}\eta$, $f''\chi\chi\eta$) the triplets and antitriplets in $\bar{\chi}$ give rise to $d = 5$ operators for proton decay. The same yukawa couplings are also responsible for the Majorana masses of the right-handed neutrinos and are certainly less constrained than the yukawa couplings $\psi\psi\zeta$ which contribute to ordinary quark and lepton masses. Values of the f 's on the order of $10^{-4} - 10^{-5}$ are sufficiently small for suppressing proton decay through $d = 5$ operators and for obtaining right-handed neutrino masses of order $10^{11} - 10^{12}$ GeV. The τ -neutrino can then be cosmologically significant. Additional suppression of these $d = 5$ operators could be obtained by some suppression of the f' and f'' couplings.

The problem of proton decay through $d = 5$ operators mediated by exchange of colour triplets and antitriplets in $\bar{\chi}$ could be more elegantly addressed by employing discrete symmetries. Let us introduce a Z_4 discrete symmetry the generator of which acts on χ and $\bar{\chi}$ as i and $-i$ respectively and leaves all other fields invariant. This symmetry forbids terms like $\bar{\chi}\bar{\chi}\eta$ and $\chi\chi\eta$ but unfortunately also the terms $\psi\psi\bar{\chi}$ responsible for right-handed neutrino Majorana masses. To deal with this problem, we further introduce a pair of $SO(10)$ singlets S, \bar{S} on which the Z_4 symmetry generator acts also as i and $-i$ and which acquire a superheavy vev as well. Now the terms $\psi\psi\bar{\chi} < S > / M_P$ (where $M_P \sim 10^{19}$ GeV is the Planck mass) are more than sufficient for right-handed neutrino Majorana masses, whereas the combined effect of these terms and the more suppressed terms $\bar{\chi}\bar{\chi}\eta < S >^2 / M_P^2, \chi\chi\eta < S >^2 / M_P^2$ give rise to sufficiently suppressed proton decay rate through $d=5$ operators involving exchange of the colour triplets and antitriplets in $\bar{\chi}$.

By employing discrete symmetries one can construct $SO(10)$ models with $\tan\beta \simeq m_t/m_b$ in which the $\chi, \bar{\chi}$ fields transforming as $126, \overline{126}$ under $SO(10)$ are replaced by spinors $\psi', \bar{\psi}'$ transforming as $16, \overline{16}$. In this case, first of all, the automatic Z_2 matter parity is broken and another matter parity has to be introduced as an additional discrete symmetry. This is easily done by assigning minus matter parity to the three ψ 's and plus matter parity to all other fields. The mechanism of suppression of $d=5$ operators by the introduction of the $210'$ representation goes through here as well. Moreover, we do not have to worry about any other $d = 5$ operators apart from the ones due to the exchange of colour triplets and antitriplets in ζ . The only

additional problem destroying the large $\tan\beta$ prediction is that there is a potential mixing between the electroweak doublets in the vector ζ (which are $SU(2)_R$ - doublets) and the electroweak doublets in the spinor ψ' (which are $SU(2)_R$ -singlets) through the $SU(2)_R$ -breaking vev $\langle \psi' \rangle$, the relevant term being $\zeta\psi' \langle \psi' \rangle$. To forbid this term we introduce a Z_3 symmetry under which the only non-invariant fields are $\psi', \bar{\psi}'$ and a gauge singlet S . These fields transform under the generator of Z_3 by multiplication with α, α^2 and α^2 respectively ($\alpha = e^{2\pi i/3}$). The singlet S acquires a vev whose natural value is $\sim M_P^{1/3} M^{2/3}$ ($M \sim 10^{16}$ GeV being the only superheavy mass scale in the model). Then the terms $\psi\psi\bar{\psi}'\bar{\psi}' \langle S \rangle / M_P$ give acceptable Majorana masses to the right-handed neutrinos while the term $\zeta\psi'\psi' \langle S \rangle^2 / M_P^2$ generates a sufficiently small $SU(2)_R$ -breaking mixing between electroweak doublets in ζ and ψ' . Therefore, again, the light electroweak higgs doublets, which must necessarily be chosen from a linear combination of ζ and θ (since ψ' does not couple to quarks and leptons), form (to a good approximation) a $SU(2)_R$ -doublet and our relation for $\tan\beta$ holds.

We presented a $SO(10)$ SUSY GUT with the asymptotic relation $\tan\beta \simeq m_t/m_b$ automatically arising from the structure of the theory. A Z_2 matter parity also emerges automatically and the significant MSSM predictions for $\sin^2\theta_w$ and α_s are retained. The $d = 5$ operators for proton decay are sufficiently suppressed. All this is achieved without imposing any global symmetries and with a relatively simple field content. Alternatives employing additional discrete symmetries were also briefly discussed.

References

1. S. Dimopoulos and H. Georgi, Nucl. Phys. B193 (1981) 150; J.Ellis, S.Kelley and D.V.Nanopoulos, Phys. Lett. B249 (1990) 441; U. Amaldi, W.de Boer and H. Furstenan, Phys. Lett. B260 (1991) 447; P.Langacker and M.X.Luo, Phys. Rev. D44 (1991) 817.
2. B. Ananthanarayan, G. Lazarides and Q. Shafi, Phys. Rev. D44 (1991) 1613; H.Arason, D.J.Castaño, B.E. Keszthelyi, S. Mikaelian, E.J.Piard, P.Ramond and B.D. Wright, Phys. Rev. Lett. 67 (1991) 2933; S. Kelley, J.L.Lopez and D.V.Nanopoulos, Phys. Lett. B272 (1992) 387.
3. G. Lazarides and C. Panagiotakopoulos, Thessaloniki Univ.preprint UT-STPD-1-94(1994).
4. N.Sakai and T.Yanagida, Nucl.Phys.B197 (1982) 533; S. Weinberg, Phys. Rev. D26 (1982) 287; J. Hisano, H. Marayama and T. Yanagida, Nucl.Phys. B402 (1993) 46.
5. J. Hisano, H. Murayama and T. Yanagida, Phys. Lett. B291 (1992) 263; K.S. Babu and S.M.Barr, Phys.Rev. D48 (1993) 5354.
6. D.G.Lee and R.N. Mohapatra, private communication.